

Opportunism vs. Cooperation: Analysis of Forwarding Strategies in Multihop Wireless Networks with Random Fading

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Abstract—A wide range of forwarding strategies have been developed for multi-hop wireless networks, considering the broadcast nature of the wireless medium and the presence of random fading that results in time-varying and unreliable transmission quality. Two recently proposed strategies are *opportunistic forwarding*, which exploits relay diversity by opportunistically selecting an overhearing relay as a forwarder, and *cooperative forwarding*, which relies on the synchronized transmissions of relays to reinforce received signal strengths. Although these strategies are well-known in the literature, there is no comprehensive comparative analysis of their network-level performance in a realistic SINR (signal-to-interference-and-noise ratio) setting with multiple network flows. In this paper, we develop Markovian models for these protocols in the case of multiple competing flows in a general network setting; we also provide recurrence relations for the special case of linear networks. We first use these models to evaluate simple small-scale networks, and find that opportunism often outperforms cooperation – a result corroborated by simulations in larger networks. We also present a fixed-point model to efficiently estimate the throughput of large networks using these models. We identify the interference resulting from the larger number of transmissions under cooperative forwarding as a cause for mitigating the potential gains achievable with cooperative forwarding.

I. INTRODUCTION

Unlike wireline networks, the broadcast nature of wireless communication allows a much richer variety of approaches for forwarding packets between a source and destination than traditional hop-by-hop forwarding along pre-specified paths. In particular, multiple nodes (in addition to the intended next-hop recipient) can overhear transmissions in a wireless network and serve as ad hoc relays to assist forwarding. Recently, two approaches have emerged that seek to exploit wireless channel characteristics when forwarding packets between source and destination in a multi-hop wireless setting:

- (1) *Opportunistic Forwarding*: Because of the broadcast nature of the wireless medium, several neighboring nodes may overhear transmissions, even if none of them is the intended next-hop or final destination. A suitable relay can often be selected opportunistically among these overhearing nodes to forward the packet downstream, until it reaches its final destination [1]–[8].
- (2) *Cooperative Forwarding*: In properly synchronized and coded networks, the signal strengths of multiple simultaneous transmissions of the same packet can be additive. Thus, if multiple nodes have received the same packet and can synchronize their forwarding transmissions of that packet, the signal strengths at downstream receivers

will be increased, thus improving the reception probability at these downstream nodes [9]–[11].

Although opportunistic forwarding and cooperative forwarding are well-known in the literature, their analysis and comparison in a network setting is rather limited. One of the challenges is to find a simple and analyzable model that realistically captures important characteristics of the wireless medium, such as signal interference strength and random fading. Most extant work either focuses on link-level analysis in one-hop networks using a complex channel fading model (e.g., [9], [10]), or multi-hop network-level analysis using a very simple channel fading model (e.g., [1], [12]). Moreover, it is also important that multiple competing flows and their interaction/interference with each other be considered and understood.

In this paper, we compare the performances of idealized and representative opportunistic and cooperative forwarding strategies under common (and realistic) assumptions. We note that the opportunistic and cooperative forwarding strategies studied in this paper are simple and the performance of both schemes can be enhanced by careful design decisions. We stress that our goal here is not to propose new protocols or investigate a specific opportunistic or cooperative transmission protocol in detail. Instead, our more fundamental goal is to characterize and understand the differences between these two approaches to forwarding in various multi-hop wireless scenarios with multiple competing flows. Our contributions are as follows:

- (1) We derive closed-form formulas for the packet reception probability in the presence of cooperative transmitters, interfering transmitters, and random fading. These results are subsequently used to study the performance (throughput) of opportunistic and cooperating forwarding strategies in multi-hop wireless networks with random fading.
- (2) We then analyze a simple n -hop linear network supporting a single flow (e.g., a wireless network along a road) under opportunistic and cooperative forwarding. We observe that in the single flow case, cooperation outperforms opportunism. This result is intuitive; in the single flow case there is no interference among packets and as there are larger number of transmitters in cooperative forwarding, the downstream packet probability reception is greater than opportunistic forwarding. Studying a single flow case in this special setting provides useful insights and helps us appreciate the results for multiple

competing flows.

- (3) We develop a Markovian model to determine the throughput achievable by opportunistic and cooperative forwarding for a general network with multiple competing flows. We analyze this model for a simple topology and show that opportunistic forwarding can achieve higher throughput than cooperative forwarding. We study larger-scale networks via simulation and observe that opportunism outperforms cooperation on average. The worse performance of cooperative forwarding can be largely attributed to the higher interference due to multiple competing flows. Lastly, we develop a fixed-point model for efficiently, but approximately computing the throughput of the Markov model, allowing performance comparisons in larger-scale networks.

Together, our results indicate that the relatively simple (and lower control overhead) opportunistic forwarding strategies is preferable to more complex cooperative counterparts in large networks with multiple competing flows. Our results also highlight the importance of considering multiple flows, since insights gained from single flow scenarios do not always carry over to the more complex multi-flow scenario, where interference among flows becomes important.

Organization: We describe the forwarding strategies in detail in Sec. III, and the wireless communication model in Sec. IV. We analyze a linear network supporting a single flow in Sec. V. For multiple flows and general topologies, we present a Markovian model in Sec. VI, which we use to study a simple diamond topology, together with simulations on random topologies. In Sec VII, we provide the fixed-point iteration for solving the Markov model.

II. RELATED WORK

The first work proposing opportunistic forwarding is [1]. Since then, several strategies have been proposed to improve the performance of opportunistic forwarding [2]–[5]. Research efforts have also theoretically analyzed the benefits of opportunism, including [6], where the authors performed a Markovian analysis to determine the expected number of network-wide link-layer transmissions needed to transfer a packet from source to destination in a wireless mesh network. [6] mostly assumes that link success probabilities are provided *a priori* and does not consider random fading in a SINR model, an important component of our models. Also, [12] provides a recursive relation for estimating the minimum number of required opportunistic transmissions. Similarly, [7] proposes an analytical model to study the performance (expected transmission count) of opportunistic routing protocols. [8] quantifies the average end-to-end delay obtained by using opportunistic schemes and demonstrates that it is about half that obtained using typical shortest path routing. None of these works consider a realistic SINR model with random fading.

A considerable amount of research has also considered cooperation in wireless networks [9], [10], [13]. [10] and [9] summarize much of this prior work in cooperative diversity and demonstrate how cooperation improves network performance. [14] is one of the few papers that describes an implementation of cooperative forwarding. It demonstrates

that by properly synchronizing sender transmissions to symbol boundaries, it is possible to outperform opportunistic routing in the absence of interference for a simple topology. Most past research on cooperation has been in the context of the physical layer, with only a few efforts exploring how cooperation interacts with higher network layers [10] and in the presence of multiple interfering flows. In [11] the authors discuss how to effectively schedule cooperative transmissions for multiple access scenarios by helping sources with poor channels to the destination use relays that have better channel quality. We note that our work differs from prior work in that we address primarily the network-layer concern (with multihop forwarding), with the goal of comparing opportunistic forwarding and cooperative forwarding – using a simple model of SINR with random fading, and in a multihop setting.

III. FORWARDING STRATEGIES

This section describes the two forwarding strategies compared in this paper – opportunistic forwarding and cooperative forwarding. We focus on generic and representative instantiations of these strategies.

- (a) *Opportunistic Forwarding:* If the packet cannot reach the destination in one hop, it is relayed by the overhearing node closest to the destination¹. This proceeds in multiple steps, until the packet reaches the destination. In the literature, there are proposals [1], [15] to address implementation details, such as how to select the appropriate relay when multiple nodes overhear the transmission². We abstract away these details, and focus on analyzing this idealized implementation in order to shed insight into the main advantage offered by opportunism - the ability to opportunistically select a relay that is closest to the destination.
- (b) *Cooperative Forwarding:* To exploit the additive property of wireless signals, multiple overhearing relays can transmit the packets towards the destination, when proper synchronization (e.g. by GPS) among multiple transmitters is feasible. This is the key innovation introduced in a cooperative strategy. We assume that a flow maintains a list of relays associated with it. When a node belonging to a list of relays of a particular flow overhears the transmission from the flow, it will be assigned as a relay. In the case of multiple network flows, we do not assume that nodes are allowed to coordinate their transmissions with other nodes that receive packets from other flows, as this would involve prohibitively high overhead. In this case, competing flows are essentially treated as interference.

A more sophisticated variant of cooperative forwarding is:

- (c) *Selective Cooperative Forwarding:* Although cooperation can reinforce signal strength, can also increase

¹In more sophisticated settings, it can be relayed by the node that has the best estimate channel condition (in some metrics [5]) to the destination. We focus on the simplest setting for our analysis.

²One solution is to use a very low-data rate, reliable out-of-band control channel to transmit the ACKs among overhearing relays [1], while the relays can be selected in a way to ensure that they can overhear ACKs among themselves [5].

the interference level to other simultaneous flows. A more refined strategy is not to assign all nodes as relays, but to instead select only a small subset of nodes that are closest to the destination or have advantageous wireless channel conditions when transmitting towards the destination. This is essentially a hybrid strategy of both opportunistic and cooperative forwarding. For convenience of analysis, we focus on a simple selective cooperative forwarding strategy that only assigns two nodes as relays that are closest to the destination among the overhearing nodes in the list of potential relays.

IV. WIRELESS COMMUNICATION MODEL

In order to compare the performance of different forwarding strategies, we use the following channel model to account for SINR and random fading. We proceed in multiple steps. Let us assume that there are C nodes in the network.

(a) *Single Transmission*: Let us first consider the simplest case with a single transmission between node i and node j ($\forall i, j = 1$ to C , $i \neq j$). Denote by $S_{i,j}$ the signal-to-noise ratio from transmitter i to receiver j :

$$S_{i,j} \triangleq \frac{|x_{i,j}|^2 P d_{i,j}^{-\alpha}}{N_0} \quad (1)$$

where N_0 is a constant background noise, $|x_{i,j}|^2$ is the Rayleigh fading coefficient (the flat fading channel is modeled as a Gaussian random process $x_{i,j}$ [16]), $d_{i,j}$ is the distance between i and j , α is the path loss exponent, and P is the transmission power at i . Note that $d_{i,j} \geq 1$ and $\alpha \geq 2$.

We assume that parameters $N_0, \alpha, P, d_{i,j}$ are constants – either known or measured a priori. On the other hand, $|x_{i,j}|^2$ is a random variable, assumed to be exponentially distributed³ with normalized mean 1. We also assume that $|x_{i,j}|^2$ is a collection of i.i.d. random variables. In this work we also assume that $|x_{i,j}|^2$ is i.i.d in different time slots. We discuss how to relax this assumption in Sec. VIII. The probability that $S_{i,j} \geq s$ (where $s > 0$) is

$$\mathbb{P}\{S_{i,j} \geq s\} = \exp\left(\frac{-sN_0}{P d_{i,j}^{-\alpha}}\right) \quad (2)$$

We model the physical layer coding scheme by assuming that a received packet can be decoded successfully when $S_{i,j} \geq \beta$ for a certain threshold β . An important quantity is the *packet reception probability* that j can successfully receive the packet from i , denoted by:

$$P_{i,j} \triangleq \mathbb{P}\{S_{i,j} \geq \beta\} \quad (3)$$

(b) *Cooperative Synchronized Transmissions*: We next consider a set of cooperative transmitters $T = \{i_1, \dots, i_m\}$ that can synchronize their transmissions such that the signal-to-noise ratio at receiver j is the sum of the individual signal-to-noise ratios from the transmitters (see Fig. 1 (a)). Note that j cannot belong to T . The total signal-to-noise ratio $S_{T,j}$ from transmitters T to j is:

$$S_{T,j} \triangleq \frac{\sum_{r \in T} |x_{r,j}|^2 P d_{r,j}^{-\alpha}}{N_0} \quad (4)$$

³In narrowband Rayleigh fading channel, the power of a signal with envelope as Rayleigh distribution is an exponential random variable [17].

Since the individual signal-to-noise ratio is an exponential random variable, the total signal-to-noise ratio is the sum of exponential random variables. Let $f_{T,j}(s)$ be the probability density function of $S_{T,j}$, which is a convolution of functions $f_{i_1,j}(s), \dots, f_{i_m,j}(s)$, defined by:

$$f_{T,j}(s) \triangleq f_{i_1,j}(s) \otimes \dots \otimes f_{i_m,j}(s) \quad (5)$$

where $f_{i,j}(s) \triangleq \frac{N_0}{P d_{i,j}^{-\alpha}} \exp\left(\frac{-sN_0}{P d_{i,j}^{-\alpha}}\right)$ is the probability density function of individual signal-to-noise ratio $S_{i,j}$.

The probability that j can successfully receive the packet from a set of transmitters T is given by $P_{T,j} \triangleq \mathbb{P}\{S_{T,j} \geq \beta\}$. Deriving a general formula for $P_{T,j}$ for an arbitrary set of transmitters T is challenging. Hence, in this paper we assume that $d_{i',j} \neq d_{i,j}$ for every pair of transmitters i, i' and any node receiver j . This significantly simplifies the proofs on the convolution of exponential distribution functions (see Lemma 1). This mild assumption, likely satisfied by real topologies, is useful to simplify the expression of $P_{T,j}$.

Lemma 1: Denote $f_{\otimes m}(s)$ as the probability density function of the sum of m independent exponential random variables with distinct means $(\lambda_1, \dots, \lambda_m)$.

$$f_{\otimes m}(s) = \left(\prod_{r=1}^m \lambda_r\right) \sum_{r=1}^m \frac{\exp(-s\lambda_r)}{\prod_{r'=1:r' \neq r}^m (\lambda_{r'} - \lambda_r)} \quad (6)$$

Proof: See [18]. ■

We remark that the general case with non-distinct values λ_r are called hypoexponential random variables [19]. There are formulas in [20], [21] for hypoexponential random variables, which appear too complicated for the analysis of network-level performance. Hence, we will rely on Lemma 1 under the assumption of distinct values of λ_r .

Lemma 2: The probability that j can successfully receive the packet from a set of transmitters T is:

$$P_{T,j} = \sum_{r \in T} \frac{\exp\left(\frac{-\beta N_0}{P d_{r,j}^{-\alpha}}\right)}{\prod_{r' \in T \setminus \{r\}} \left(1 - \left(\frac{d_{r',j}}{d_{r,j}}\right)^\alpha\right)} \quad (7)$$

Proof: Based on Lemma 1, see [22]. ■

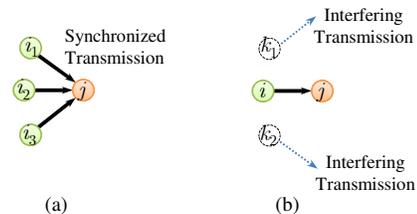


Fig. 1. (a) Cooperative synchronized transmission vs. (b) Competing interfering transmission.

(c) *Competing Interfering Transmissions*: Lemma 2 only considers the case of cooperative synchronized transmitters. To address the case of competing interfering transmissions (e.g., Fig. 1 (b)), let I be the set of simultaneously competing transmitters. The signal-to-interference-and-noise ratio $S_{i,j}^I$ from transmitter i to receiver j in the presence of a set of interfering transmitters I is defined as:

$$S_{i,j}^I \triangleq \frac{|x_{i,j}|^2 P d_{i,j}^{-\alpha}}{N_0 + \sum_{k \in I} |x_{k,j}|^2 P d_{k,j}^{-\alpha}} \quad (8)$$

It is clear that $i \neq j$ and that i and j cannot belong to I . The probability that j can successfully receive the packet from transmitter i is given by $P_{i,j}^I \triangleq \mathbb{P}\{S_{i,j}^I \geq \beta\}$, which can be obtained from the following lemma.

Lemma 3:

$$P_{i,j}^I = \sum_{k \in I} \frac{\exp\left(\frac{-\beta N_0}{P d_{i,j}^{-\alpha}}\right)}{\left(1 + \beta \left(\frac{d_{i,j}}{d_{k,j}}\right)^\alpha\right) \prod_{k' \in I \setminus \{k\}} \left(1 - \left(\frac{d_{k',j}}{d_{k,j}}\right)^\alpha\right)} \quad (9)$$

Proof: Omitted due to space constraints. See [22]. ■

(d) *Mixed Cooperative & Interfering Transmissions:* Last, we consider the general case with a set of cooperative transmitters T and a set of simultaneously competing transmitters I . The signal-to-interference-and-noise ratio $S_{T,j}^I$ from a set of cooperative transmitters T to j in the presence of a set of interfering transmitters I is:

$$S_{T,j}^I \triangleq \frac{\sum_{r \in T} |x_{r,j}|^2 P d_{r,j}^{-\alpha}}{N_0 + \sum_{k \in I} |x_{k,j}|^2 P d_{k,j}^{-\alpha}} \quad (10)$$

Note that $T \cap I = \emptyset$ and j cannot belong to T or I . The probability that j can successfully receive the packet from a set of cooperative transmitters T in spite of interfering transmitters I is given by $P_{T,j}^I \triangleq \mathbb{P}\{S_{T,j}^I \geq \beta\}$, which can be obtained by the following lemma, derived using Lemmas 2-3.

Lemma 4:

$$P_{T,j}^I = \sum_{r \in T} \sum_{k \in I} \frac{\exp\left(\frac{-\beta N_0}{P d_{r,j}^{-\alpha}}\right)}{\left(1 + \beta \left(\frac{d_{r,j}}{d_{k,j}}\right)^\alpha\right) \prod_{r' \in T \setminus \{r\}} \left(1 - \left(\frac{d_{r',j}}{d_{r,j}}\right)^\alpha\right) \prod_{k' \in I \setminus \{k\}} \left(1 - \left(\frac{d_{k',j}}{d_{k,j}}\right)^\alpha\right)}$$

V. NETWORKS WITH SINGLE FLOW

Sec. IV provides single-transmission/reception models for wireless networks with random fading. We now use this model to construct simple recurrence relations for source-destination paths and compare the performance of opportunistic to cooperative forwarding strategies in a linear network. We observe that in the single flow case (where there is no network interference), the throughput provided by cooperative forwarding is greater than that provided by opportunistic forwarding. In the following, we consider the single packet case⁴ - the source sends no new packet until the packet reaches the destination.

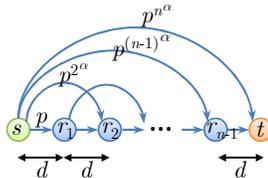


Fig. 2. An n -hop linear network.

A. Opportunistic Forwarding

In Fig. 2, we consider only one flow in a n -hop linear network, where s is the source, t is the destination, and $s = r_0, r_1, \dots, r_{n-1}, r_n = t$ are the relays. Assume that the distance between r_{i-1} and r_i ($\forall i, j = 1$ to n) in the linear network is d , and denote by $p \triangleq \exp\left(\frac{-\beta N_0}{P d^{-\alpha}}\right)$ the packet reception probability for a transmission over one hop. Hence,

⁴We will discuss the multiple packet case in Sec.VIII

the probability that i can successfully transmit packets to j when they are n hops apart is given by:

$$P_{i,j} = \exp\left(\frac{-\beta N_0}{P (nd)^{-\alpha}}\right) = p^{n\alpha} \quad (11)$$

For convenience of analysis, we assume α is an integer.

Suppose the distance between i and j is nd , and the distance between j and interfering transmitter k is $m'd$.

$$P_{i,j}^k = \frac{\exp\left(\frac{-\beta N_0}{P (nd)^{-\alpha}}\right)}{1 + \beta \left(\frac{n}{m'}\right)^\alpha} = \frac{p^{n\alpha}}{1 + \beta \left(\frac{n}{m'}\right)^\alpha} \quad (12)$$

There are two quantities of interest. One quantity is the throughput of the linear network. Denote by $N_{\text{op}}[n]$ the expected number of transmissions required by opportunistic forwarding to reach the destination from the source in a n -hop linear network. We obtain:

$$N_{\text{op}}[1] = \frac{1}{p}$$

$$N_{\text{op}}[n] = p^{n\alpha} + \sum_{i=1}^{n-1} \prod_{j=i+1}^n (1 - p^{j\alpha}) p^{i\alpha} (1 + N_{\text{op}}[n - i]) + \prod_{j=1}^n (1 - p^{j\alpha}) (1 + N_{\text{op}}[n]) \quad (13)$$

To write a recursive equation for $N_{\text{op}}[n]$ (13), we have to consider three cases: 1) With probability $p^{n\alpha}$, the source can reach the destination in one transmission. 2) With probability $\prod_{j=i+1}^n (1 - p^{j\alpha}) p^{i\alpha}$, the source can reach the node that is $(n - i)$ hops away from the destination in one transmission, from which the expected number of transmissions to reach the destination is $N_{\text{op}}[n - i]$. 3) Otherwise, with probability $\prod_{j=1}^n (1 - p^{j\alpha})$, the source cannot reach any other nodes.

Because there is only a single flow, the throughput is:

$$T_{\text{op}}[n] = \frac{1}{N_{\text{op}}[n]} \quad (14)$$

Another quantity of interest denoted by $H_{\text{op}}[n]$ is the average number of hops traversed in one transmission, given that the destination is n hops away. We obtain the recurrence equation:

$$H_{\text{op}}[1] = p$$

$$H_{\text{op}}[n] = np^{n\alpha} + (1 - p^{n\alpha})H_{\text{op}}[n - 1] \quad (15)$$

We can solve $H_{\text{op}}[n]$ in closed form.

Lemma 5:

$$H_{\text{op}}[n] = \sum_{j=1}^n j p^{j\alpha} \left(\prod_{\ell=j+1}^n (1 - p^{\ell\alpha}) \right) \quad (16)$$

When $n \geq 2$,

$$H_{\text{op}}[n] = p + 2p^{2\alpha} - p^{1+2\alpha} + 3p^{3\alpha} + O(p^{1+3\alpha}) \quad (17)$$

Proof: Omitted due to space constraints. See [22]. ■

Theorem 1: The throughput is upper bounded by:

$$T_{\text{op}}[n] \leq \frac{1}{n} H_{\text{op}}[n] \quad (18)$$

Proof: Omitted due to space constraints. See [22]. ■

In general, we observe that the upper bound is tight (see Fig. 3).

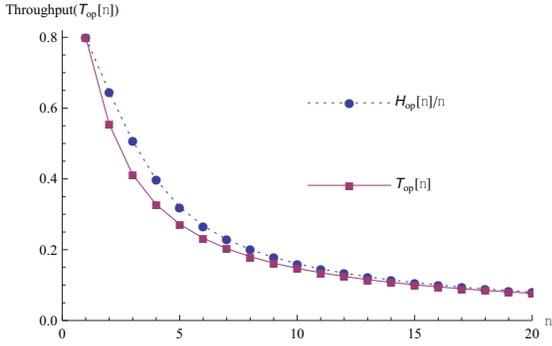


Fig. 3. ($p = 0.8$) The throughput $T_{\text{op}}[n]$ is tightly bounded by $\frac{1}{n}H_{\text{op}}[n]$.

B. Cooperative Forwarding

Next, we consider cooperative forwarding using all over-hearing relays to transmit the packets to the destination.

We consider the idealized case of *perfect* cooperative forwarding, where we can use all the relays between the source and farthest overhearing relay in the linear network for cooperative forwarding. Thus, in Fig. 2, if r_k overhears the packet, then we assume all relays r_1, \dots, r_{k-1} also overhear the packets. We aim at bounding the gap between opportunistic forwarding and cooperative forwarding. Hence, it suffices to consider perfect cooperative forwarding in order to establish an upper bound on this gap.

Assuming perfect cooperation, let $H_{\text{co}}[n]$ be the average number of hops reached in one transmission by cooperative forwarding, given that the destination is n hops away.

Theorem 2: The expected number of hops a packet can reach in one time slot by cooperative forwarding is related to that of opportunistic forwarding by:

$$H_{\text{co}}[n] = O(\sqrt{n}) \cdot H_{\text{op}}[n] \quad (19)$$

Proof: Omitted due to space constraints. See [22]. ■

In this section, we have seen that cooperative forwarding provides higher performance (at most sub-linear, i.e., \sqrt{n} improvement) than opportunistic forwarding in the single-flow, linear-network case. As we will see in subsequent sections, where we consider multiple competing flows within the network, transmission interference among flows (which is *not* present in the single flow case) becomes a critical factor. This will mitigate the advantages of cooperative forwarding found in this section, suggesting that the relative advantages of opportunistic and collaborative forwarding depend strongly on network topology and assumptions about traffic flows.

VI. NETWORKS WITH MULTIPLE FLOWS

Having studied the single flow case for a linear network in the previous section, we next consider a general setting with an arbitrary network topology and multiple flows. The major difference between the single flow and the multiple flow case is the increased interference due to competing flows. We formulate Markovian models for analyzing the different forwarding strategies in the multiple flow scenario, using the packet reception probabilities from Sec. IV. Using these models, we study a simple topology, and show that opportunistic forwarding can outperform cooperative forwarding in the absence of inter-flow cooperation. For more general networks we use simulation and observe that opportunism outperforms

cooperation on average. Thus, we conclude that interference mitigates the potential gains of cooperative forwarding.

A. Opportunistic Forwarding

First, we present the Markov model for opportunistic forwarding in a general network topology and multiple flows; we then evaluate this model for a simple diamond network. We denote a set of flows by \mathcal{F} . Each flow $f \in \mathcal{F}$ has a list of participating nodes denoted by $\mathcal{P}_f = (v_{\text{s}(f)}, v_1, \dots, v_{\text{d}(f)})$, where $v_{\text{d}(f)}$ is the destination and $v_{\text{s}(f)}$ is the source. Each succeeding node in \mathcal{P}_f (e.g., v_i) has a higher priority than its preceding nodes (i.e., v_j for all $j < i$) for forwarding the packet, until the packet reaches $v_{\text{d}(f)}$. Formally, we denote “ $v_i \succ_f v_j$ ” to represent that v_i has a higher priority than v_j in \mathcal{P}_f .

We denote the state of the network as $r \triangleq (r_f \in \mathcal{P}_f : f \in \mathcal{F})$, where r_f is the active relay for flow f for the next forwarding operation. Recall that $P_{T,j}^I$ is the packet reception probability at j from a set of cooperative transmitters T in the presence of interfering transmitters I . We denote by $\mathbf{P}_{r,r'}$ the state transition probability from state r to state r' , where ($r'_f \succ_f r_f$ or $r'_f = r_f$) and $r'_f \neq v_{\text{d}(f)}$, for at least one flow $f \in \mathcal{F}$. Let $r_{-f} \triangleq \{r_{f'} : f' \in \mathcal{F} \setminus \{f\}\}$. We obtain:

$$\mathbf{P}_{r,r'} \triangleq \prod_{f \in \mathcal{F}} P_{r_f, r'_f}^{r-f} \cdot \prod_{v \in \mathcal{P}_f : v \succ_f r'_f} (1 - P_{r_f, v}^{r-f}) \quad (20)$$

Namely, $\mathbf{P}_{r,r'}$ is the packet reception probability that every flow f can receive a packet from r_f to r'_f , subject to the condition that the set of succeeding nodes $\{v \in \mathcal{P}_f : v \succ_f r'_f\}$ that cannot receive the packet.

Recall that we assume that the flow’s source transmits a new packet after the successful delivery of a packet to the the flow’s destination. Therefore when a flow reaches state $r_f = v_{\text{d}(f)}$ (i.e., the packet reaches the destination), the state transition in the next time step will correspond to the states reachable from the source with their respective probabilities.

We remark that Eqn. (20) contains two prohibited cases: 1) two packets cannot be received by the same receiver at the same time; and 2) a node cannot be receiving and transmitting at the same time. Because we assume $\beta > 1$, Eqn. (20) will give zero transition probability for the above two cases.

The stochastic behavior of the network is characterized by the Markov chain defined by state transition probability $\mathbf{P}_{r,r'}$ for every pair of states (r, r') . We then can evaluate the stationary distribution $\pi(r)$ for each state of the network r , which satisfies the following balance equation:

$$\sum_{r''} \pi(r'') \cdot \mathbf{P}_{r'',r} = \sum_{r'} \pi(r') \cdot \mathbf{P}_{r,r'} \quad (21)$$

subject to $\sum_r \pi(r) = 1$.

The throughput of each flow f , $T_{\text{op}}(f)$, is given by:

$$T_{\text{op}}(f) = \sum_{r: r_f = v_{\text{d}(f)}} \pi(r) \quad (22)$$

B. Cooperative Forwarding

For cooperative forwarding, we denote the state of the network as R , where $R_f \subseteq \mathcal{P}_f$ is a set of cooperative transmitters of flow f . Let $R_{-f} \triangleq \bigcup_{f' \in \mathcal{F} \setminus \{f\}} R_{f'}$. The state

transition probability $\mathbf{P}_{R,R'}$, where $R_f \subseteq R'_f$ for at least one $f \in \mathcal{F}$ and $v_{d(f)} \notin R_f$, is given by:

$$\mathbf{P}_{R,R'} \triangleq \prod_{f \in \mathcal{F}} \prod_{v \in R'_f \setminus R_f} P_{R_f,v}^{R-f} \cdot \prod_{v' \in \mathcal{P}_f \setminus R'_f} (1 - P_{R_f,v'}^{R-f}) \quad (23)$$

Similarly, Eqn. (23) also contains the prohibited cases, to ensure that (1) two packets cannot be received by the same receiver at the same time; and (2) a node cannot be receiving and transmitting at the same time.

The stationary distribution is denoted by $\pi(R)$, and the throughput of flow f , $T_{\text{co}}(f)$, is given by:

$$T_{\text{co}}(f) = \sum_{R,R':v_{d(f)} \in R'_f} \pi(R) \cdot \mathbf{P}_{R,R'} \quad (24)$$

C. Selective Cooperative Forwarding

Selective cooperative forwarding only assigns two closest nodes to the destination that currently have a copy of the packet as relays. We again denote the state of the network as R , where $R_f \subseteq \mathcal{P}_f$ is a set of potential transmitters of flow f that have received the packet. The two nodes $r_1, r_2 \in R_f$ are selected, such that $r_1 \succ_f r_2 \succ_f r$ for all $r \in R_f \setminus \{r_1, r_2\}$. Hence, we denote the two selected relays by a set $r(R_f)$.

Let $I_f(r(R)) \triangleq \bigcup_{f' \in \mathcal{F} \setminus \{f\}} r(R_{f'})$. The state transition probability $\mathbf{P}_{R,R'}$, where $R_f \subseteq R'_f$ for at least one $f \in \mathcal{F}$ and $v_{d(f)} \notin R_f$, is given by:

$$\mathbf{P}_{R,R'} \triangleq \prod_{f \in \mathcal{F}} \prod_{v \in R'_f \setminus R_f} P_{r(R_f),v}^{I_f(r(R))} \cdot \prod_{v' \in \mathcal{P}_f \setminus R'_f} (1 - P_{r(R_f),v'}^{I_f(r(R))})$$

D. Analysis of a Simple Diamond Network

Using the Markov models defined in Secs. VI-A and VI-B, we compare the performance of opportunistic forwarding and cooperative forwarding with two flows in the diamond network depicted in Fig. 4 (a). There are two flows: $s_1 \rightarrow t_1$ and $s_2 \rightarrow t_2$. Relay r can contribute to either flow, depending on if it can overhear the flow. We assume $\beta > 1$, and by Eqn. (10) a node can receive a packet from only one flow at a time.

The forwarding operations for opportunistic forwarding and cooperative forwarding respectively generate the Markov chains in Fig. 5. Each state corresponds to a subset of transmitters that can forward the packet. In opportunistic forwarding, relay r can forward the packet for a flow during a time slot, provided that it received a packet previously. In cooperative forwarding, both source and relay will participate in forwarding. Hence, both Markov chains have the same structure, but different state transition probabilities.

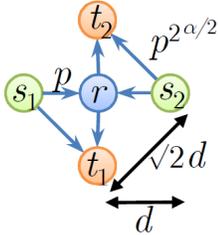


Fig. 4. Simple Diamond Network

(a) *Opportunistic Forwarding*: The stationary distribution is:

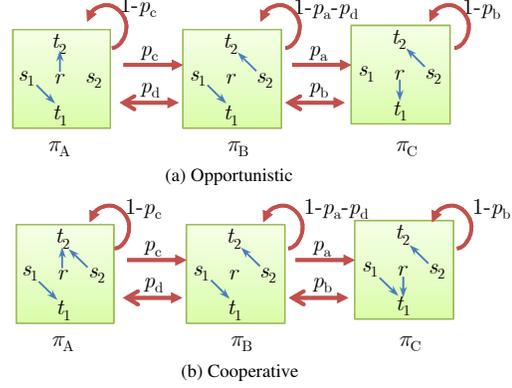


Fig. 5. Markov chains for the forwarding operations in a diamond network.

$$\pi_A = \frac{p_b p_d}{p_a p_c + p_b p_c + p_b p_d}, \quad \pi_B = \frac{p_b p_c}{p_a p_c + p_b p_c + p_b p_d}$$

$$\pi_C = \frac{p_a p_c}{p_a p_c + p_b p_c + p_b p_d} \quad (25)$$

where the state transition probabilities can be expressed in terms of the packet reception probabilities:

$$p_a = (1 - P_{s_1, t_1}^{s_2}) P_{s_1, r}^{s_2}, \quad p_d = p_a$$

$$p_b = P_{r, t_1}^{s_2}, \quad p_c = p_b \quad (26)$$

In Table I, we compute all the packet reception probabilities using Lemmas 2-4.

Consider $\alpha = 2$. The throughput of flow $s_1 \rightarrow t_1$ using opportunistic forwarding is given by:

$$T_{\text{op}}^{s_1 \rightarrow t_1} \triangleq \pi_A P_{s_1, t_1}^r + \pi_B P_{s_1, t_1}^{s_2} + \pi_C P_{r, t_1}^{s_2}$$

$$= \frac{p^2 (p(2+3\beta+\beta^2) + 4(1+3\beta+2\beta^2)) - p^3 (2+\beta) - p^2 (2+4\beta)}{2(1+2\beta)((1+\beta)^2 - p^3(2+\beta) + p(2+3\beta+\beta^2))} \quad (27)$$

(b) *Cooperative Forwarding*: The state transition probabilities can be expressed in terms of the packet reception probabilities by Lemmas 2-4 (see Table I):

$$p_a = (1 - P_{s_1, t_1}^{s_2}) P_{s_1, r}^{s_2}, \quad p_d = p_a$$

$$p_b = P_{\{s_1, r\}, t_1}^{s_2}, \quad p_c = p_b \quad (28)$$

The throughput of flow $s_1 \rightarrow t_1$ using cooperative forwarding is given by:

$$T_{\text{co}}^{s_1 \rightarrow t_1} \triangleq \pi_A P_{s_1, t_1}^{\{r, s_2\}} + \pi_B P_{s_1, t_1}^{s_2} + \pi_C P_{\{s_1, r\}, t_1}^{s_2}$$

$$= \frac{p^2 (-3 + 2p^2 - 4\beta)}{(1 + 2\beta) (2p^2 - 3(1 + \beta))} \quad (29)$$

Lemma 6: The throughput of opportunistic forwarding is superior to that of cooperative forwarding:

$$T_{\text{op}}^{s_1 \rightarrow t_1} \geq T_{\text{co}}^{s_1 \rightarrow t_1} \quad (30)$$

Proof: See the Appendix. ■

We also plot the throughput of a flow using opportunistic forwarding and cooperative forwarding in the case of two competing flows in Fig. 6. This corroborates our intuition that cooperation can degrade performance due to the increased amount of interference generated by the larger number of simultaneous transmitters.

E. Simulations for Random Networks

Having studied a simple diamond network via a Markovian analysis, we next use simulation to study larger network settings. As we will see, the insights gained in the small

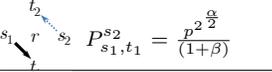
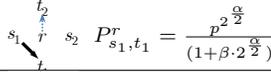
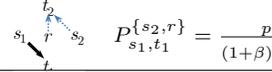
		
$P_{s_1, t_1}^{s_2} = \frac{p^{2\frac{\alpha}{2}}}{(1+\beta)}$	$P_{s_1, t_1}^r = \frac{p^{2\frac{\alpha}{2}}}{(1+\beta \cdot 2^{\frac{\alpha}{2}})}$	$P_{s_1, t_1}^{\{s_2, r\}} = \frac{p^{2\frac{\alpha}{2}}}{(1+\beta)(1+2^{\frac{\alpha}{2}})}$
$P_{r, t_1}^{s_2} = \frac{p}{(1+\beta \cdot 2^{-\frac{\alpha}{2}})}$	$P_{s_1, r}^{s_2} = \frac{p}{(1+\beta)}$	$P_{\{s_1, r\}, t_1}^{s_2} = \frac{p^{2\frac{\alpha}{2}}}{(1+\beta)(1-2^{-\frac{\alpha}{2}})} + \frac{p}{(1+\beta \cdot 2^{-\frac{\alpha}{2}})(1-2^{-\frac{\alpha}{2}})}$

TABLE I

PACKET RECEPTION PROBABILITIES FOR TRANSMISSIONS IN BOLD ARROWS, UNDER THE INTERFERENCE OF THE TRANSMISSIONS IN DASHED ARROWS.

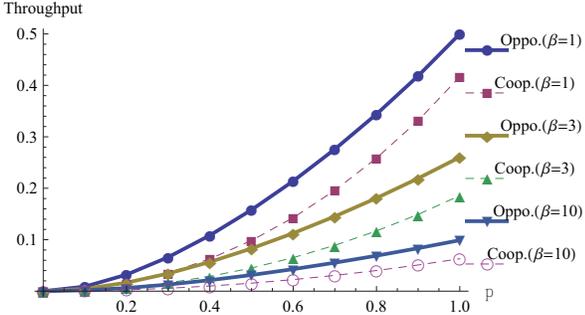


Fig. 6. The throughput of opportunistic forwarding and cooperative forwarding in the simple diamond network.

scale setting generally apply in this more general setting. We consider 50 nodes uniformly distributed in a 100×100 area. We select 4 distinct source-destination pairs (referred to as a ‘configuration’) at random from the 50 nodes. We simulate the link quality between different pairs of nodes for every time slot using the Rayleigh fading channel model. The simulation begins by all 4 sources transmitting packets. A node is able to receive a packet if the SINR between the transmitter and itself is above a threshold. The opportunistic and cooperative routing protocols govern the nodes that transmit packets in the next time slot. When a packet reaches the corresponding destination, the source starts transmitting a new packet. We conduct this simulation for 5000 time slots and keep track of the number of packets received at the destination to calculate the throughput. We refer to the simulation of a given ‘configuration’ as a ‘run’.

Recall that our earlier results revealed that interference among different flows played an important factor in determining performance. Thus, when presenting throughput comparisons in this section, we would like to quantify such interference among packets flowing from source to destination along ‘paths’ between given sets of source-destination pairs. Note, however, that there is no well-defined notion of a deterministic ‘path’ along which packets flow for either opportunistic or cooperative forwarding. Thus we characterize the level of interference among flows by taking 10 points equidistantly-spaced between a flow’s source and destination for each flow. Let $\mathcal{L}(f)$ denote the set of these 10 points for flow f . We consider the distance between all pairs of such points i, j in the following manner:

$$\text{Intf-Metric} \triangleq \sum_{f \in \mathcal{F}} \sum_{f' \in \mathcal{F} \setminus \{f\}} \sum_{i \in \mathcal{L}(f)} \sum_{j \in \mathcal{L}(f')} d_{i,j}^{-\alpha} \quad (31)$$

Intf-Metric provides a coarse measure of the interference (as caused by the nearness of potentially interfering nodes for different flows). Higher value of Intf-Metric indicates a greater amount of interference in the network. In Fig. 7 we

plot the difference in throughput between the opportunistic and cooperative strategies versus the Intf-Metric for a β of 4. The figure is obtained by conducting 100 ‘runs’, each time with a different ‘configuration’. Points above the line drawn at Throughput Difference=0 indicate the cases where opportunism performs better than cooperation while the points below the line depict the opposite. The results in Fig. 7 indicate that on average the performance of opportunism is better than that of cooperation and this is true for a wide range of Intf-Metric values.

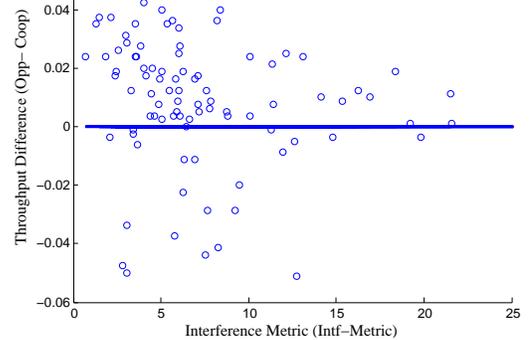


Fig. 7. Simulation results of throughput difference in random networks.

VII. FIXED-POINT MODEL

Sec. VI provided a Markovian model of multiple flows in a general network setting. However, the number of states increases exponentially with the number of flows in this model. Hence, evaluating the stationary distribution of the model quickly becomes intractable. Thus, in this section we introduce a fixed-point model to simplify the evaluation. We will show that the throughput obtained from the fixed-point model is a lower bound to the actual throughput of the Markov model. In order to establish this model as a lower bound, we assume that the sets of participating nodes among distinct flows are disjoint (i.e., $\mathcal{P}_f \cap \mathcal{P}_{f'} = \emptyset$).

Our approach is to model each flow independently and capture their dependence by accounting for interference between flows within each flow model. This gives rise to a set of fixed-point equations for the stationary distributions, which can be obtained via efficient iterative methods.

A. Opportunistic Forwarding

The state of a flow $f \in \mathcal{F}$ is specified by the active relay $r \in \mathcal{P}_f$ that overhears the transmission and has the highest priority among the overhearing participating nodes. We next describe a set of fixed-point equations for individual flows.

First, suppose that the stationary distribution of a node $j \in \mathcal{P}_f$ to become an active relay is given by $\hat{\pi}_f(j)$. Then, the

expected interference to j from all other flows w.r.t. stationary distributions $\hat{\pi}_{-f} \triangleq \{\hat{\pi}_{f'} : f' \in \mathcal{F} \setminus \{f\}\}$ becomes:

$$\begin{aligned} \hat{I}_j^f(\hat{\pi}_{-f}) &= \sum_{f' \in \mathcal{F} \setminus \{f\}} \sum_{i \in \mathcal{P}_{f'}} \hat{\pi}_{f'}(j) \cdot \mathbb{E}[|x_{i,j}|^2] \cdot \text{Pd}_{i,j}^{-\alpha} \\ &= \sum_{f' \in \mathcal{F} \setminus \{f\}} \sum_{i \in \mathcal{P}_{f'}} \hat{\pi}_{f'}(j) \cdot \text{Pd}_{i,j}^{-\alpha} \end{aligned} \quad (32)$$

where the fading coefficient $|x_{i,j}|^2$ is an i.i.d. exponential random variable with normalized mean 1. Note that $\hat{I}_j^f(\hat{\pi}_{-f})$ does not depend on $\hat{\pi}_f$, but only on $\{\hat{\pi}_{f'} : f' \in \mathcal{F} \setminus \{f\}\}$.

Suppose that the interference from other flows remains stationary and has distribution $\hat{\pi}_{-f}$. Then the packet reception probability that j can successfully receive the packet from i in flow f w.r.t. $\hat{\pi}_{-f}$ is given by:

$$\hat{P}_{i,j}^f(\hat{\pi}_{-f}) \triangleq \mathbb{P}\left\{ \frac{|x_{i,j}|^2 \text{Pd}_{i,j}^{-\alpha}}{\text{N}_0 + \hat{I}_j^f(\hat{\pi}_{-f})} \geq \beta \right\} = \exp\left(\frac{-\beta(\text{N}_0 + \hat{I}_j^f(\hat{\pi}_{-f}))}{\text{Pd}_{i,j}^{-\alpha}} \right) \quad (33)$$

Next, we focus on the Markov model of individual flow. In such a model, we denote by $\hat{\mathbf{P}}_{r,r'}^f(\hat{\pi}_{-f})$ the state transition probability from an active relay $r \in \mathcal{P}_f$ to another active relay $r' \in \mathcal{P}_f$ such that $(r' \succ_f r$ or $r' = r)$ and $r \neq v_{d(f)}$, w.r.t. $\hat{\pi}_{-f}$:

$$\hat{\mathbf{P}}_{r,r'}^f(\hat{\pi}_{-f}) \triangleq \hat{P}_{r,r'}^f(\hat{\pi}_{-f}) \cdot \prod_{v \in \mathcal{P}_f: v \succ_f r'} \left(1 - \hat{P}_{r,v}^f(\hat{\pi}_{-f})\right) \quad (34)$$

Denote the stationary distribution of this model as $\hat{\pi}_f$. It satisfies the following balance equations for all $r \in \mathcal{P}_f$:

$$\sum_{r'' \in \mathcal{P}_f} \hat{\pi}_f(r'') \cdot \hat{\mathbf{P}}_{r'',r}^f(\hat{\pi}_{-f}) = \sum_{r' \in \mathcal{P}_f} \hat{\pi}_f(r') \cdot \hat{\mathbf{P}}_{r,r'}^f(\hat{\pi}_{-f}) \quad (35)$$

subject to $\sum_{v \in \mathcal{P}_f} \hat{\pi}_f(v) = 1$.

Eqns. (32)-(35) form a set of fixed-point equations for $(\hat{\pi}_f : f \in \mathcal{F})$. Solving the fixed-point $(\hat{\pi}_f : f \in \mathcal{F})$ can be achieved by an iterative method. We first assume a certain distribution $(\hat{\pi}_f^0 : f \in \mathcal{F})$. Then we obtain $\hat{\pi}_f^1$ from Eqns. (32)-(35) w.r.t. $\hat{\pi}_f^0$, for all $f \in \mathcal{F}$. We repeat the process for t steps, until $\hat{\pi}_f^t$ has a small deviation from $\hat{\pi}_f^{t-1}$.

The throughput of the fixed-point model is defined by:

$$\hat{\mathbf{T}}_{\text{op}}(f) = \hat{\pi}_f(v_{d(f)}) \quad (36)$$

Theorem 3: The throughput obtained from the fixed-point model is a lower bound to the actual throughput of the Markov model:

$$\mathbf{T}_{\text{op}}(f) \geq \hat{\mathbf{T}}_{\text{op}}(f) \quad (37)$$

Proof: See the Appendix. ■

B. Cooperative Forwarding

The fixed-point model for cooperative forwarding is similar to that of opportunistic forwarding. But the state of a flow is specified by the set of cooperative transmitters $R \subseteq \mathcal{P}_f$. By Lemma 2, the packet reception probability that j can successfully receive the packet from the set of cooperative transmitters R in a flow f is given by:

$$\hat{P}_{R,j}^f(\hat{\pi}_{-f}) = \sum_{r \in R} \frac{\exp\left(\frac{-\beta(\text{N}_0 + \hat{I}_j^f(\hat{\pi}_{-f}))}{\text{Pd}_{r,j}^{-\alpha}}\right)}{\prod_{r' \in R \setminus \{r\}} \left(1 - \left(\frac{d_{r',j}}{d_{r,j}}\right)^\alpha\right)} \quad (38)$$

Note that the state of flow is R , a subset of cooperative transmitters. Then the stationary distribution of a node $j \in \mathcal{P}_f$ is given by:

$$\hat{\pi}_f(j) = \sum_{R \subseteq \mathcal{P}_f: j \in R} \hat{\pi}_f(R) \quad (39)$$

In the Markov model of an individual flow f , the state transition probability $\mathbf{P}_{R,R'}$ from $R \subseteq \mathcal{P}_f$ to $R' \subseteq \mathcal{P}_f$ such that $R \subseteq R'$ and $v_{d(f)} \notin R$, is defined by:

$$\hat{\mathbf{P}}_{R,R'}^f(\hat{\pi}_{-f}) \triangleq \prod_{v \in R' \setminus R} \hat{P}_{R,v}^f(\hat{\pi}_{-f}) \cdot \prod_{v' \in \mathcal{P}_f \setminus R'} \left(1 - \hat{P}_{R,v'}^f(\hat{\pi}_{-f})\right) \quad (40)$$

To solve the fixed-point $(\hat{\pi}_f : f \in \mathcal{F})$, we rely on a similar iterative approach as for the case of opportunistic forwarding. The throughput obtained from the fixed-point model can be shown as a lower bound to the actual throughput of the Markov model, using the same argument as in Theorem 3.

C. Selective Cooperative Forwarding

The case of selective cooperative forwarding is similar to basic cooperative forwarding, except with a modification to consider the two best relays instead all relays. Specifically, let the two best relays be $r_1, r_2 \in R$ for flow f , such that $r_1 \succ_f r_2 \succ_f r$ for all $r \in R \setminus \{r_1, r_2\}$. We denote the two selected relays by a set $r_f(R)$. The packet reception probability that j can successfully receive the packet from the set of cooperative transmitters R in a flow f is given by $\hat{P}_{r_f(R),j}^f$. And the stationary distribution of a node $j \in \mathcal{P}_f$ is given by:

$$\hat{\pi}_f(j) = \sum_{R \subseteq \mathcal{P}_f: j \in r_f(R)} \hat{\pi}_f(R) \quad (41)$$

The state transition probability $\mathbf{P}_{R,R'}$ from $R \subseteq \mathcal{P}_f$ to $R' \subseteq \mathcal{P}_f$ such that $R \subseteq R'$ and $v_{d(f)} \notin R$, is defined by:

$$\hat{\mathbf{P}}_{R,R'}^f(\hat{\pi}_{-f}) \triangleq \prod_{v \in R' \setminus R} \hat{P}_{r_f(R),v}^f(\hat{\pi}_{-f}) \cdot \prod_{v' \in \mathcal{P}_f \setminus R'} \left(1 - \hat{P}_{r_f(R),v'}^f(\hat{\pi}_{-f})\right) \quad (42)$$

To solve the fixed-point $(\hat{\pi}_f : f \in \mathcal{F})$, we rely on a similar iterative approach as for the case of opportunistic forwarding. The throughput obtained from the fixed-point model can be shown as a lower bound to the actual throughput of the Markov model, using the same argument as in Theorem 3.

D. Comparison of Fixed Point and Simulation

We compare the performance of our models with the simulation results. For these simulations we consider a 5×5 grid topology and consider opportunistic forwarding and selective cooperative forwarding. The simulation procedure is similar to the one outlined in Sec. VI-E. For the model we iteratively solve the fixed point equations in Sec. VII-A and VII-C for the opportunistic and selective cooperative forwarding strategies. Results in Fig. 8 are obtained considering 5 parallel flows, each moving vertically downwards in the grid. Flows are given ids ranging from 1 to 5 starting from one end of the grid to the other. As expected, we find that for both schemes Flows 1 and 5 have maximum and comparable throughput as they experience the minimum amount of interference from other flows. Flow 3 has the minimum throughput because it is situated in the middle and experiences maximum interference.

Moreover the throughput of opportunism is greater than cooperation. This is primarily due to increased interference in the cooperative case because of greater number of transmitters. We note that although the throughput obtained by our fixed-point model is lower than that obtained by simulation the relative ordering between opportunistic and cooperative forwarding is preserved.

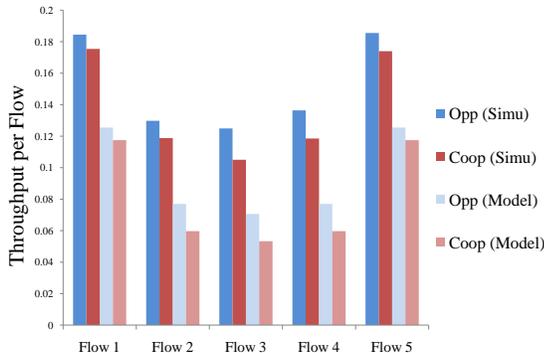


Fig. 8. Comparison between simulation and fixed-point model

VIII. CONCLUSION AND FUTURE WORK

This paper has used modeling and analysis to investigate the performance benefits of using opportunism and cooperation forwarding in wireless networks. Rather than proposing new protocols or investigating the performance of specific opportunistic or cooperative transmission protocols, our goal instead was to compare the performance of idealized and representative opportunistic and cooperative forwarding strategies using generic models and under common realistic assumptions.

We began with a single flow linear network, and observed that cooperation outperforms opportunism. We then considered the case of more general network topologies with multiple flow and observed that unlike the linear network case, opportunism outperformed cooperation on average. We identified the interference resulting from the larger number of transmissions under cooperative forwarding as a cause for mitigating the potential gains achievable with cooperative forwarding.

There are numerous avenues for future research. In this paper we considered the single packet case i.e., there is only one packet in transit for each source-destination pair. Our current research is aimed at extending our analysis to cases where multiple packets are pipelined for each flow. Here we believe our single-packet throughput analysis can form the basis for a pipelined analysis for protocols that control transmission to provide a “guard zone” around each packet within a flow, ensuring that intra-flow interference among packet transmissions does not occur. In this case, the model reduces to serial, pipelined instances of the models considered in this paper. A potential complexity here will be to determine the size of the guard zone. Too large guard zone will decrease the pipelining efficiency, whereas too small guard zone would result in high interference. An on-going extension is to optimize the guard zone and study performance of different forwarding strategies.

We assumed that fading is i.i.d distributed in different time slots. This assumption will hold true only for fast fading where the coherence time is smaller than the duration of a time slot. In case of slow fading where the i.i.d assumption will not hold,

cooperation will have additional benefits over opportunism because of multiple transmitters. The fading correlation of $x_{i,j}$ is a Bessel function [16] and we plan to incorporate the effect of correlation into the Markov model in our future work.

A longer term challenge is to compare opportunistic and cooperative forwarding in the presence of competing flows, with optimized (centrally or distributed) scheduling. The practical, but important, question of the overhead needed to achieve this coordination in practice, and whether this additional complexity is warranted by the increase in performance is also a question for future research.

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IX. APPENDIX

A. Proof of Lemma 6

Proof: By substitution, we obtain:

$$\frac{\mathbb{T}_{\text{op}}^{s_1 \rightarrow t_1}}{\mathbb{T}_{\text{co}}^{s_1 \rightarrow t_1}} = \frac{(2p^2 - 3(1+\beta))(p(2+3\beta+\beta^2) + 4(1+3\beta+2\beta^2) - p^3(2+\beta) - p^2(2+4\beta))}{2(-3+2p^2-4\beta)((1+\beta)^2 - p^3(2+\beta) + p(2+3\beta+\beta^2))}$$

Because $0 \leq p \leq 1$,

$$\begin{aligned} & (p(2+3\beta+\beta^2) + 4(1+3\beta+2\beta^2) - p^3(2+\beta) - p^2(2+4\beta)) - ((1+\beta)^2 - p^3(2+\beta) + p(2+3\beta+\beta^2)) \\ &= 3 + 10\beta + 7\beta^2 - 2p^2(1+2\beta) \geq 1 + 6\beta + 7\beta^2 \\ & (2p^2 - 3(1+\beta)) - 2(-3+2p^2-4\beta) \\ &= 3 - 2p^2 + 5\beta \geq 1 + 5\beta \end{aligned}$$

Therefore, we obtain $\frac{\mathbb{T}_{\text{op}}^{s_1 \rightarrow t_1}}{\mathbb{T}_{\text{co}}^{s_1 \rightarrow t_1}} \geq 1$ ■

B. Proof of Theorem 3

Proof: First, we consider the actual Markov chain of multiple flows defined in Sec. VI-A. We recall that the stationary distribution of the actual Markov chain of multiple flows is π over the set $\{r : r_f \in \mathcal{P}_f\}$. Note that π is the fixed-point to Eqns. (20)-(21). Let \mathcal{R} be the random set of active relays that are transmitting for the flows. Let the random total interference level to node j be:

$$I_j(\mathcal{R}) \triangleq \sum_{r \in \mathcal{R} \setminus \{j\}} |x_{r,j}|^2 \cdot Pd_{r,j}^{-\alpha} \quad (43)$$

Let $P_{i,j}^f$ be the packet reception probability from node i to j , where j belongs to flow f . Since the set of active relays is random, $P_{i,j}^f$ is a random variable, with expected value

$$\begin{aligned} \mathbb{E}[P_{i,j}^f] &= \mathbb{E}\left[\mathbb{P}\left\{\frac{|x_{i,j}|^2 Pd_{i,j}^{-\alpha}}{N_0 + I_j(\mathcal{R})} \geq \beta\right\}\right] \\ &= \mathbb{E}\left[\exp\left(\frac{-\beta(N_0 + I_j(\mathcal{R}))}{Pd_{i,j}^{-\alpha}}\right)\right] \end{aligned} \quad (44)$$

The throughput of a flow can be obtained by averaging over time. By the ergodicity of the Markov model, it is equivalent to averaging over stationary distribution. Let the stationary distribution of each state j of flow f be $\pi_f(j) \triangleq \sum_{r:r_f=j} \pi(r)$, and $\mathbf{1}(r \rightarrow v_{d(f)})$ be the indicator function that there is a state transition from r to $v_{d(f)}$ for flow f at a timeslot.

$$\begin{aligned} \mathbb{T}_{\text{op}}(f) &= \sum_{r \in \mathcal{P}_f \setminus \{v_{d(f)}\}} \pi_f(r) \cdot \mathbb{E}_{\pi}[\mathbf{1}(r \rightarrow v_{d(f)})] \\ &= \sum_{r \in \mathcal{P}_f \setminus \{v_{d(f)}\}} \pi_f(r) \cdot \mathbb{E}_{\pi}\left[P_{r,v_{d(f)}}^f \cdot \prod_{v \in \mathcal{P}_f: v \succ_f r} (1 - P_{r,v}^f)\right] \\ &= \sum_{r \in \mathcal{P}_f \setminus \{v_{d(f)}\}} \pi_f(r) \cdot \mathbb{E}_{\pi}[P_{r,v_{d(f)}}^f] \cdot \prod_{v \in \mathcal{P}_f: v \succ_f r} (1 - \mathbb{E}_{\pi}[P_{r,v}^f]) \end{aligned}$$

The last equality is due to the assumption of independent random fading among pairs of nodes (i.e., fading coefficient $|x_{i,j}|^2$ is an i.i.d. random variable for any pair of nodes i, j).

Second, recall that $(\hat{\pi}_f : f \in \mathcal{F})$ is the fixed-point to Eqns. (32)-(35), which is also a fixed-point to Eqn. (35) and the following equation:

$$\hat{\mathbf{P}}_{r,r'}(\hat{\pi}) \triangleq \prod_{f \in \mathcal{F}} \hat{P}_{r_f,r'_f}^f(\hat{\pi}_{-f}) \cdot \prod_{v \in \mathcal{P}_f: v \succ_f r'} (1 - \hat{P}_{r_f,v}^f(\hat{\pi}_{-f})) \quad (45)$$

Note that Eqns. (45) & (35) are comparable to Eqns. (20)-(21) for stationary distribution π .

The throughput can be given by:

$$\begin{aligned} \hat{\mathbb{T}}_{\text{op}}(f) &= \sum_{r \in \mathcal{P}_f \setminus \{v_{d(f)}\}} \hat{\pi}_f(r) \cdot \mathbf{P}_{r,v_{d(f)}}^f(\hat{\pi}_{-f}) \\ &= \sum_{r \in \mathcal{P}_f \setminus \{v_{d(f)}\}} \hat{\pi}_f(r) \cdot \hat{P}_{r,v_{d(f)}}^f(\hat{\pi}_{-f}) \cdot \prod_{v \in \mathcal{P}_f: v \succ_f r} (1 - \hat{P}_{r,v}^f(\hat{\pi}_{-f})) \end{aligned}$$

Next, we compare $\mathbb{E}_{\hat{\pi}}[P_{r,r'}^f]$ and $\hat{P}_{r,r'}^f(\hat{\pi}_{-f})$ under a certain distribution $\hat{\pi}$ over r . Since $\exp(-x)$ is a convex function, by Jensen's inequality,

$$\begin{aligned} \mathbb{E}_{\hat{\pi}}[P_{i,j}^f] &= \mathbb{E}_{\hat{\pi}}\left[\exp\left(\frac{-\beta(N_0 + I_j(\mathcal{R}))}{Pd_{i,j}^{-\alpha}}\right)\right] \\ &= \geq \exp\left(\frac{-\beta(N_0 + \mathbb{E}_{\hat{\pi}}[I_j(\mathcal{R})])}{Pd_{i,j}^{-\alpha}}\right) \end{aligned} \quad (46)$$

Then, by the definition of $P_{i,j}^f(\hat{\pi}_{-f})$, we have $\mathbb{E}_{\hat{\pi}}[I_j(\mathcal{R})] = \hat{I}_j^f(\hat{\pi}_{-f})$. This implies

$$\mathbb{E}_{\hat{\pi}}[P_{i,j}^f] \geq P_{i,j}^f(\hat{\pi}_{-f}) \quad (47)$$

If $\mathbb{E}_{\hat{\pi}}[\hat{P}_{i,j}^f] \geq \hat{P}_{i,j}^f(\hat{\pi}_{-f})$ for any pair of nodes i, j and any distribution $\hat{\pi}$, then every forwarding operation carries a lower packet reception probability in the latter case. Hence, the latter case must have decreased throughput under the respective fixed-point:

$$\mathbb{T}_{\text{op}}(f) \geq \hat{\mathbb{T}}_{\text{op}}(f) \quad (48)$$

■