A Markov chain model for coarse timescale channel variation in an 802.16e wireless network

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ABSTRACT

We are developing a finite-state Markov Chain (MC) channel model to capture wireless channel variations due to shadowing, which occur at a time scale of seconds. The Markov Chain is constructed by partitioning the entire range of shadowing into a finite number of intervals. We determine the MC transition matrix in two ways: (i) via a parsimonious modeling approach in which shadowing effects are modeled as a log-normally distributed random variable affecting the received power, and the transition probabilities are derived as functions of the variance and autocorrelation function of shadowing; (ii) via an empirical approach, in which the MC transition matrix is calculated by directly measuring the changes in signal strengths collected in a 802.16e (WiMAX) network. We present a validation of the abstract model by comparing its steady state and transient performance predictions with those computed using the empirically derived transition matrix.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication

General Terms

Theory, Measurement

1. INTRODUCTION

In this short paper we describe a Markov chain model, and its validation, to model the effects of shadowing on the received signal strength. This shadowing model can be used in analyzing performance of wireless network protocols (e.g., for route adaptation, or for video transmission) that adapt their behavior in response to link-level changes at the timescale of seconds. The underlying physical channel model assumes that the variation in received signal strength due to shadowing is a lognormally-distributed random variable with zero mean[4] and has an exponential autocorrelation function, which in turn implies that shadowing follows a First Order Autoregressive AR(1) process. These assumptions (which we test via measurements in a WiMAX (802.16e) network) provide the theoretical framework for adopting a Markov chain model for capturing the effect of shadowing on received power. We divide the entire range of shadowing into a finite number of intervals. We consider two approaches for determining the transition matrix of the Markov chain: *(i) a parsimonious model-based approach.* in which we use the log-normal distribution of received signal strength and exponential autocorrelation to derive expressions for the transition probabilities; this approach is parsimonious as the transition probabilities are dependent only on the variance (σ^2) and the exponent (ρ) of the exponential autocorrelation function of shadowing; *(ii) an empirical approach* in which transition probabilities are determined from the empirical frequency (taken from measurement traces) of moving from one state to another. We perform signal strength measurements in a WiMAX (802.16e) network and validate the assumptions of the MC model, and its predictions, using these measurements.

The log-normal nature of shadowing has been reported in [4, 7]; [1] is the seminal paper describing exponentially distributed autocorrelation. The correlation properties of shadowing for an indoor channel have been studied in [2, 5]. [3] empirically characterizes the received power over a wireless channel for the outdoor-to-outdoor and indoor-to-indoor environment. Our work differs in its focus on developing and validating a Markov chain model for shadowing, assuming the log-normal distribution and exponential autocorrelation of shadowing.

2. SHADOWING BASED CHANNEL MODEL

In this section we describe the MC channel model and derive its transition matrix. Let d, α, d_0 be the transmitter-to-receiver separation, the path loss coefficient and the close-in reference distance respectively. The received power $P_r(d)$ in [dBm] considering log-normal shadowing [4] is given by

$$P_r(d)[dBm] = \bar{P}_r(d_0) + 10\alpha log\frac{d}{d_0} + X \quad (1)$$

where $\bar{P}_r(d_0)$ is the average received power at the reference distance d_0, the second term the logarithmic dependence on distance and X is the shadowing which is a zero-mean Gaussian random variable with variance σ^2 in [dB]. Therefore (1) describes the effect of shadowing on the received power.

As noted earlier, shadowing (in dB) is assumed to be $N(0, \sigma^2)$ while both its spatial and temporal autocorrelation functions are assumed to be exponential [1]. Let X_i

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and X_{i+n} be the shadowing samples at time i and $i+n$ respectively. The temporal autocorrelation coefficient between X_i and X_{i+n} is given by,

$$\rho_n = \frac{E[X_i X_{i+n}]}{\sigma^2} = e^{-\frac{n\delta t}{\tau}} \quad (2)$$

Let $\rho = e^{-\frac{\delta t}{\tau}}$, and so $\rho_n = \rho^n$. The exponential autocorrelation function implies that the random process is a first-order autoregressive AR(1) process, and therefore

$$X_i = \rho X_{i-1} + (1-\rho)e_i \quad (3)$$

where e_i is white noise and is $\sim N(0, \sigma_e^2)$. Further e_i and X_{i-1} are independent of each other. X_i being an AR(1) process also implies that shadowing is a Markovian process [6] as the distribution of X_i given X_{i-1} is the same as X_i given X_{i-1}, X_{i-2}....,X_1. This fact is also clear from (3) as X_i is only dependent on X_{i-1}.

To construct the MC model, the entire range of shadowing is partitioned into N intervals, $(A_0, A_1....A_N)$, where A_0 and A_N correspond to $-\infty$ and ∞ respectively. Let Y_i denote that the X value is between A_{i-1} and A_i. Therefore $Y_i's$ denote the states of the Markov Chain. We now determine the transition probability P_{ij},(i.e. the probability of transitioning from range Y_i to range Y_j) by the parsimonious model-based and empirical approaches.

2.1 Model-based Transition Matrix

We begin by stating the following lemma, the proof of which is omitted due to lack of space.

LEMMA 1. *Two consecutive shadowing samples X_k and X_{k-1} are Jointly Gaussian*

X_k and X_{k-1} being Jointly Gaussian (Lemma 1) implies that $X_{k+1}|X_k \sim N(\rho x_k, \sigma^2(1-\rho^2))$. Moreover, recall that $X_k \sim N(0, \sigma^2)$. To calculate the transition probability P_{ij} analytically, we make use of the above mentioned Gaussian assumptions, to obtain:

$$\begin{aligned} P_{ij} &= P(X_{k+1} \in Y_j | X_k \in Y_i) \\ &= \frac{\int_{Y_i}(\int_{Y_j} f_{X_{k+1}|X_k}(x_2|x_1)dx_2) f_{X_k}(x_1)dx_1}{\int_{Y_i} f_{X_k}(x_1)dx_1} \quad (4) \end{aligned}$$

The distributions of $X_{k+1}|X_k$ and X_k being Gaussian, $\int_{Y_j} f_{X_{k+1}|X_k}(x_2|x_1)dx_2$ and $\int_{Y_i} f_{X_k}(x_1)dx_1$ can be calculated on basis of error functions. The value of P_{ij} can then be numerically calculated as the integral in the numerator can be easily solved using any mathematical package like MATLAB, once the values of ρ and σ have been determined.

2.2 Empirical Transition Matrix

We also determine the transition matrix empirically, by performing signal strength measurements. We first extract the shadowing values by eliminating the deterministic distance dependent path loss. We then determine the states of the MC to which each of the shadowing values correspond to, and compute the empirical probabilities of transitioning between these states from the traces.

3. VALIDATING THE MODEL

We collected data under varying levels of user mobility (pedestrian and vehicular) for experiments carried out over a 802.16e (WiMAX) network at WINLAB in New Jersey.

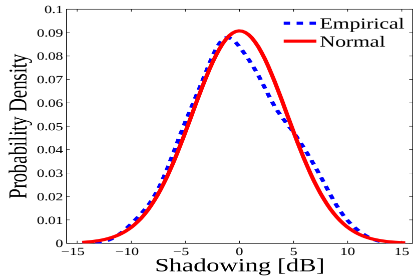


Figure 1: WiMAX: Shadow Samples follow a Normal Distribution

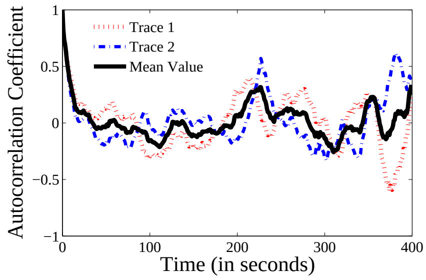


Figure 2: WiMAX: Autocorrelation of Shadow Samples (Vehicular Mobility)

Channel quality measurements were taken by continuously transmitting data from a base station and receiving them on a laptop. The WiMAX measurements were carried out outdoors for pedestrian and vehicular mobility cases. Distance variation from the base station outdoors was captured by using a GPS device attached to the laptop, with received power samples collected one second apart from each another. The samples are averaged over a time window of 50 ms to eliminate any fast fading effects. The shadowing samples were extracted by observing the deviation of the received power samples from the log distance relation.

Normality Testing of Shadow Samples. We adopt the *Kolmogorov-Smirnov* goodness of fit test to determine the normality of shadowing for the traces collected. Let σ_{sam}^2 denote the variance of the collected samples for any trace considered. The null hypothesis is the following: The samples are drawn from a normal distribution having mean 0 and variance σ_{sam}^2. Our tests failed to reject the null hypothesis at any acceptable level of significance for both the vehicular and pedestrian traces. The smoothened probability distribution obtained for one of the vehicular traces using the kernel density estimation method and the corresponding normal distribution are shown in Figure 1. The standard deviation for this trace is 4.4 dB.

Autocorrelation FunctionTesting. The temporal autocorrelation function of shadowing for two different vehicular and pedestrian traces along with the mean is shown in Figures 2 and 3 respectively. We observe that the autocorrelation function does not follow an exponential distribution when the traces are considered individually.

4. RESULTS

In the previous section we tested 1) normality and 2) autocorrelation (exponential form) of shadowing and observed that the shadowing process is not Markovian in reality. In this section we construct the Markov Chain and compare the predicted model-based and empirical steady state perfor-

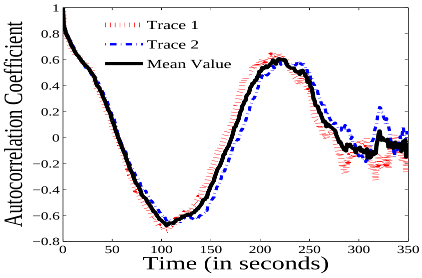
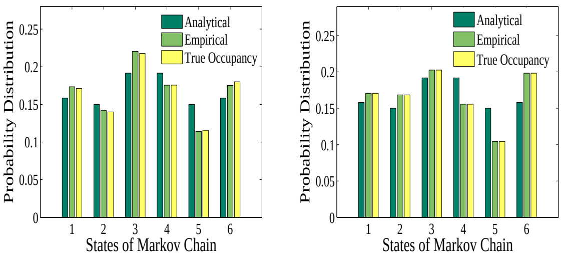


Figure 3: WiMAX: Autocorrelation of Shadow Samples (Pedestrian Mobility)

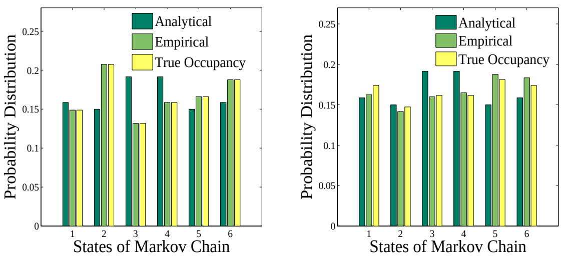


(a) Trace 1 (b) Trace 2

Figure 4: Comparison of observed shadowing-state occupancies with the analytical and empirical steady state occupancies of the Markov Chain (Vehicular Mobility)

mance with the observed shadowing-state occupancies. We construct the Markov chain by dividing shadowing into the following intervals: $\{-\infty, -\sigma_{sam}, -\frac{\sigma_{sam}}{2}, 0, \frac{\sigma_{sam}}{2}, \sigma_{sam}, \infty\}$ and determine the model-based and empirical transition matrices for the vehicular and pedestrian mobility traces. Tables 1 and 2 show the model-based and empirical transition matrix respectively for the vehicular trace in Figure 1.

Figures 4 and 5 compare the empirically observed shadowing-state occupancies (True Occupancy), as well as those predicted by the parsimonious MC model (Analytical) and the MC model using the observed transition probabilities (Empirical), for the vehicular and pedestrian traces. Overall, both Figures 4 and 5 show good agreement in the model-predicted and observed steady state shadowing values, in spite of the lack of exponential autocorrelation evidenced in Figures 2 and 3. We are currently performing a transient analysis to further explore the impact of this assumption.



(a) Trace 1 (b) Trace 2

Figure 5: Comparison of observed shadowing-state occupancies with the analytical and empirical steady state occupancies of the Markov Chain (Pedestrian Mobility)

0.6433	0.2334	0.0983	0.0211	0.0023	0.0001
0.2471	0.3290	0.2811	0.1169	0.0235	0.0024
0.0815	0.2202	0.3374	0.2519	0.0915	0.0175
0.0175	0.0915	0.2519	0.3374	0.2202	0.0815
0.0024	0.0235	0.1169	0.2811	0.3290	0.2471
0.0001	0.0023	0.0211	0.0983	0.2334	0.6433

Table 1: Model-based Transition Matrix

0.6883	0.2078	0.0909	0	0	0.0130
0.2381	0.3651	0.3016	0.0794	0	0.0159
0.0619	0.1649	0.4948	0.1856	0.0515	0.0412
0.0380	0.0633	0.2152	0.4304	0.1519	0.1013
0	0.0192	0.0577	0.2885	0.3846	0.2500
0	0.0244	0.0488	0.0854	0.1829	0.6585

Table 2: Empirical Transition Matrix

5. CONCLUSION

In this short paper, we developed and validated a finite-state Markov Chain (MC) channel model to capture wireless channel variations due to shadowing. The MC transition matrix in two ways: (i) via a parsimonious modeling approach in which shadowing effects are modeled as a log-normally distributed random variable affecting the received power, and the transition probabilities are derived as functions of the variance and autocorrelation function of shadowing; (ii) via an empirical approach, in which the MC transition matrix is calculated by directly measuring the changes in signal strengths collected in an 802.16e (WiMAX) network. The model validation showed that the assumption that the variation in received signal strength due to shadowing had a lognormally-distributed random variable was a good one, but that the assumption of an exponential autocorrelation function was violated. Nonetheless, the MC model showed good agreement between the model-predicted and observed steady state values of shadowing.

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